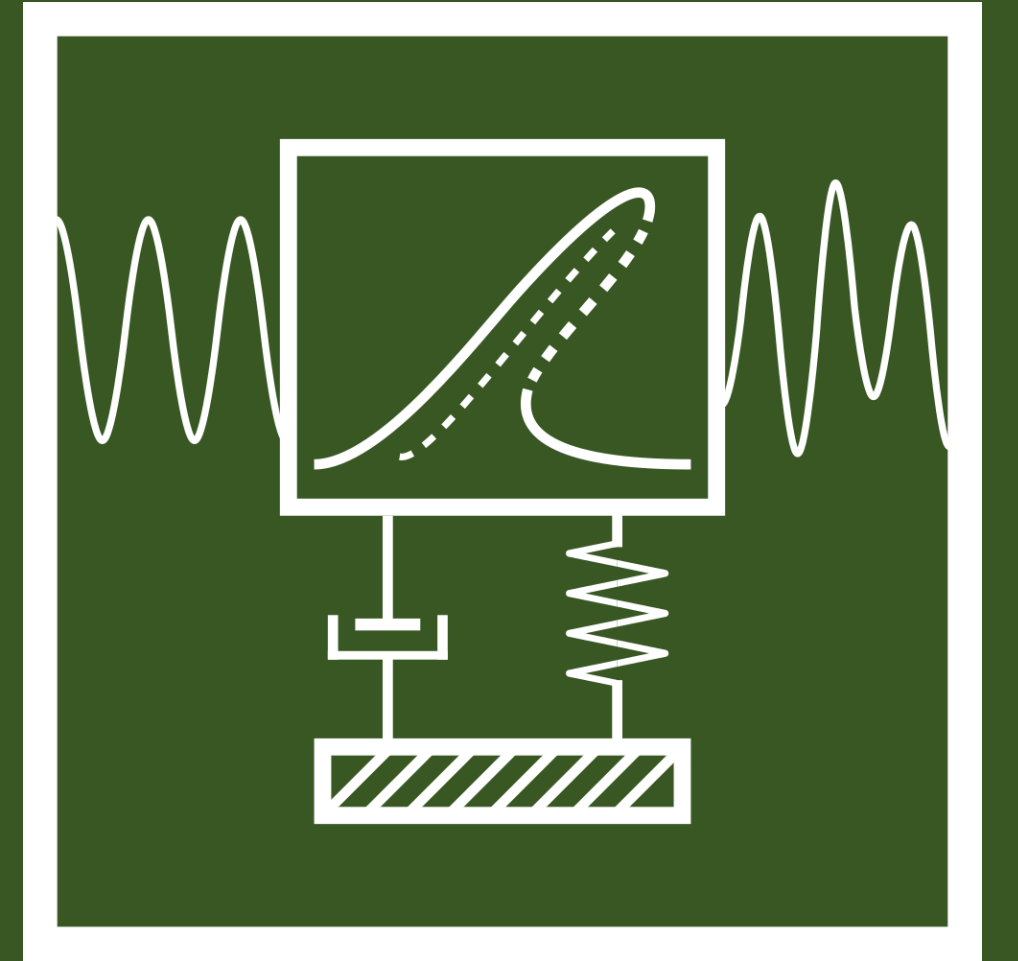


H-rank Based Impulsive Stabilization Algorithm for the Fractional Difference Logistic Map

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A Abstract

□ An impulse-based stabilization scheme for the fractional difference logistic map (FDLM) is presented in this research. It is expressed as:

$$x_{n+1} = x_0 + \sum_{j=1}^{n+1} G_{j-1}^{\nu} (ax_{n-j+1}(1-x_{n-j+1}) - x_{n-j+1}), \quad j = 0, 1, \dots,$$

where $G_0^{\nu} = 1$, $G_j^{\nu} = \left(1 - \frac{1-\nu}{j}\right) G_{j-1}^{\nu}$, $j = 1, 2, \dots$. Parameter ν is the fractionality parameter [2].

□ An impulse-based control technique that avoids short oscillatory transients immediately following the control impulse is designed for the FDLM with a memory horizon that reaches the initial condition.

□ However, continuous application of the proposed control scheme appears to cause a drift in the coordinate of the unstable period-1 orbit.

□ An impulse-based control scheme is preferred over continuous feedback control schemes because it is minimally invasive and, consequently, preserves the system model by requiring small, sparse, and instantaneous impulses to achieve finite-time stabilization of the unstable period-1 orbit of the FDLM

Aim: To create an impulse-based stabilization scheme that is able to stabilize the unstable-period 1 orbit of the FDLM.

01 Methods

□ The naïve stabilization scheme (see Fig. 1) is insufficient to achieve finite-time stabilization of the unstable period-1 orbit because the memory horizon of the FDLM reaches the initial condition inclusively [3].

□ Hankel ranks (H-ranks for short) are a measure of complexity used to evaluate the complexity of each initial condition that starts from x_0 , and are used in the H-rank based impulsive stabilization scheme.

□ The H-rank based impulsive stabilization scheme tracks at which step k the trajectory x_k exceeds the tolerance corridor placed around the unstable period-1 coordinate $x_* = 1 - \frac{1}{a}$ with a width of δ . Once the tolerance corridor is breached, we evaluate possible initial conditions using the H-rank method to find the initial condition x_k for finite-time stabilization [3].

02 Results

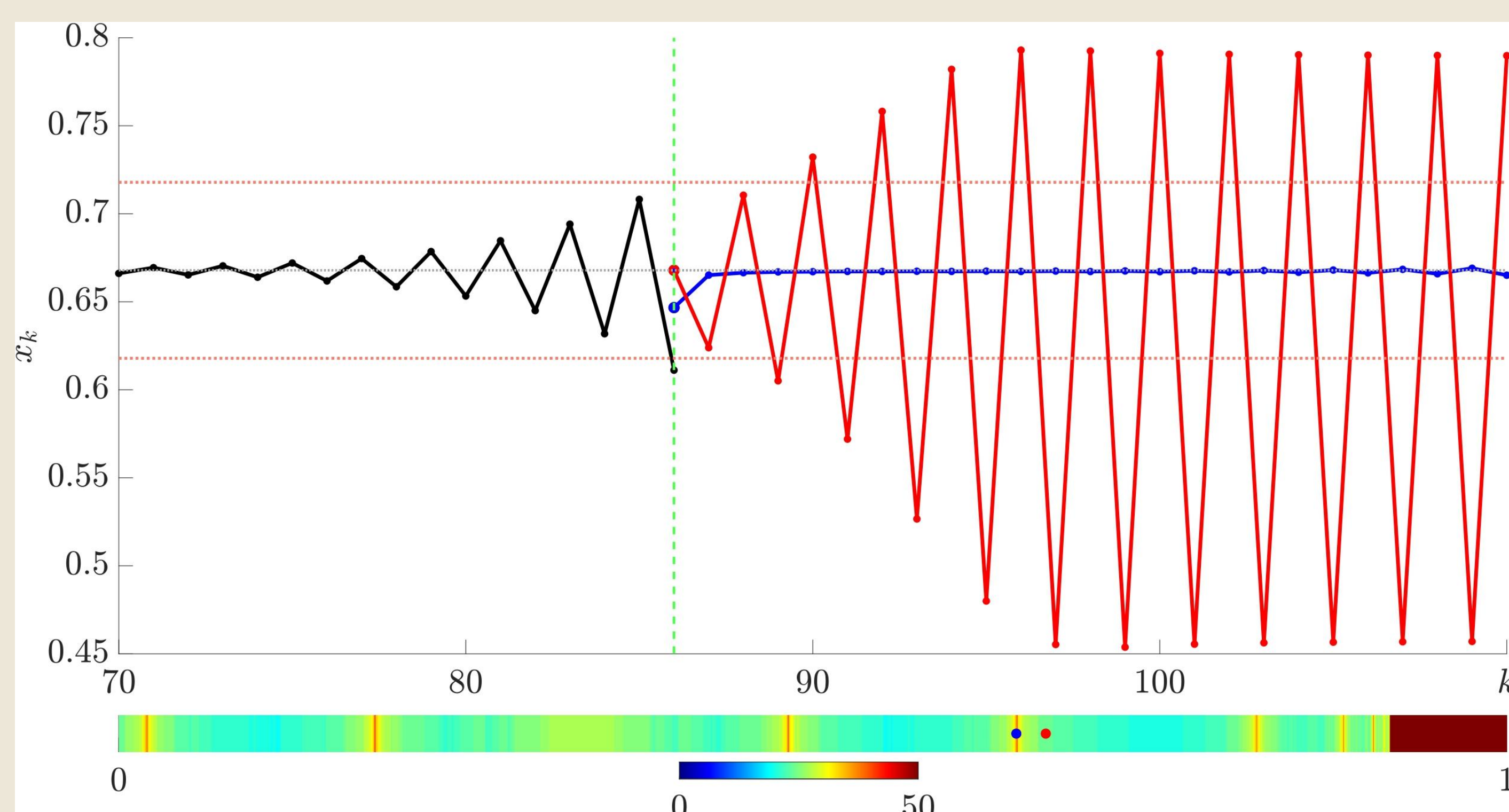


Fig 1. Naïve stabilization scheme for the FDLM [3]. The black line corresponds to the trajectory before stabilization, red – if stabilization is not applied, blue – if stabilization is applied. The green vertical dashed line is the step k where the initial trajectory breaches the tolerance corridor δ (marked in two horizontal red dotted lines). Below the stabilization graph is the H-rank pattern graph. The red and blue dots represent stabilization initial conditions x_k for naïve stabilization and H-rank based stabilization respectively.

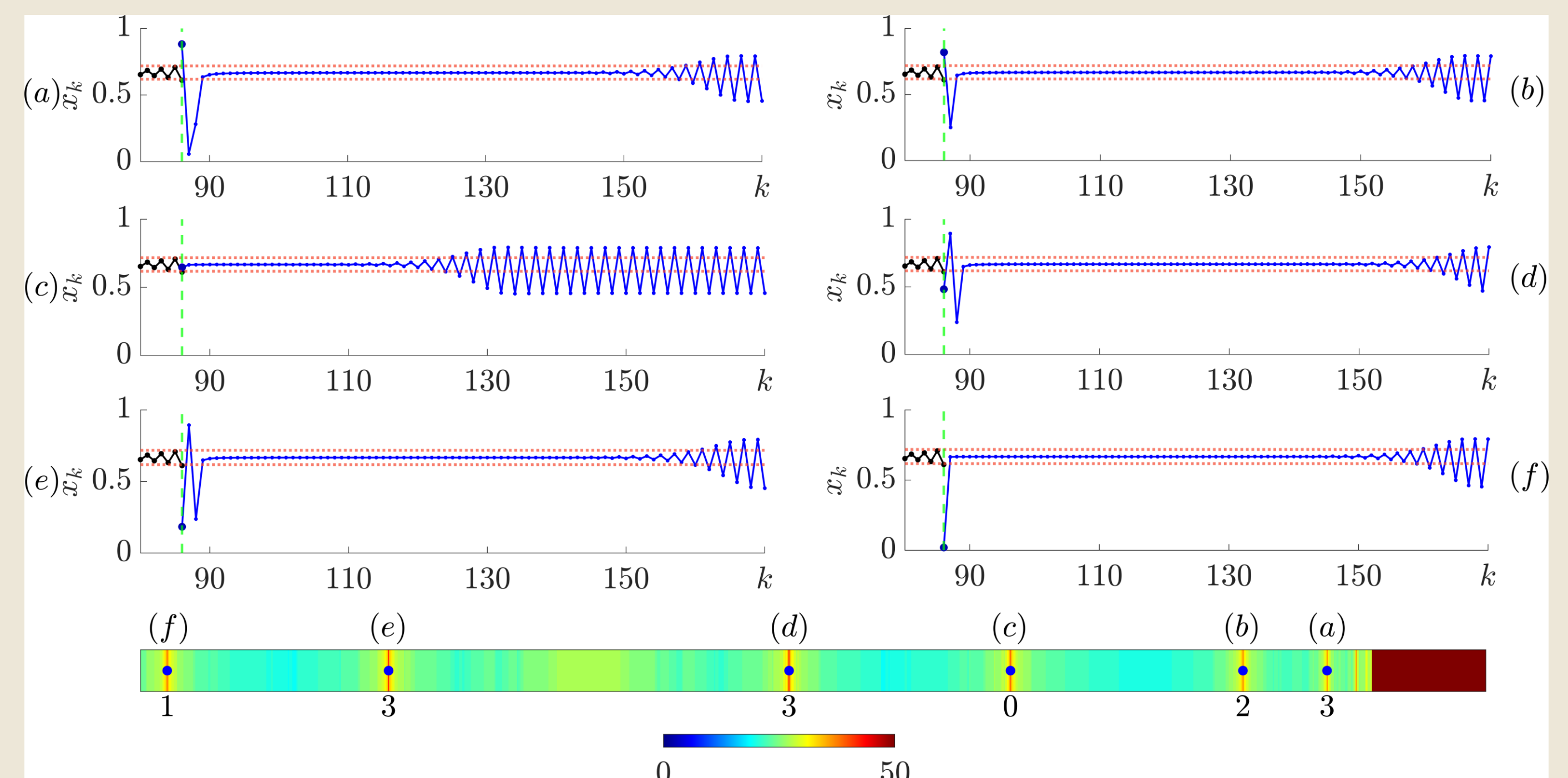


Fig 2. H-rank based impulsive stabilization attempts. Panels (a) to (f) illustrate several possible initial conditions for stabilization, of which only panel (c) does not breach the tolerance corridor (marked in two red horizontal dotted lines) following the application of the stabilization impulse. Below the stabilization graph is the H-rank pattern graph where the stabilization initial conditions x_k are marked accordingly [3].

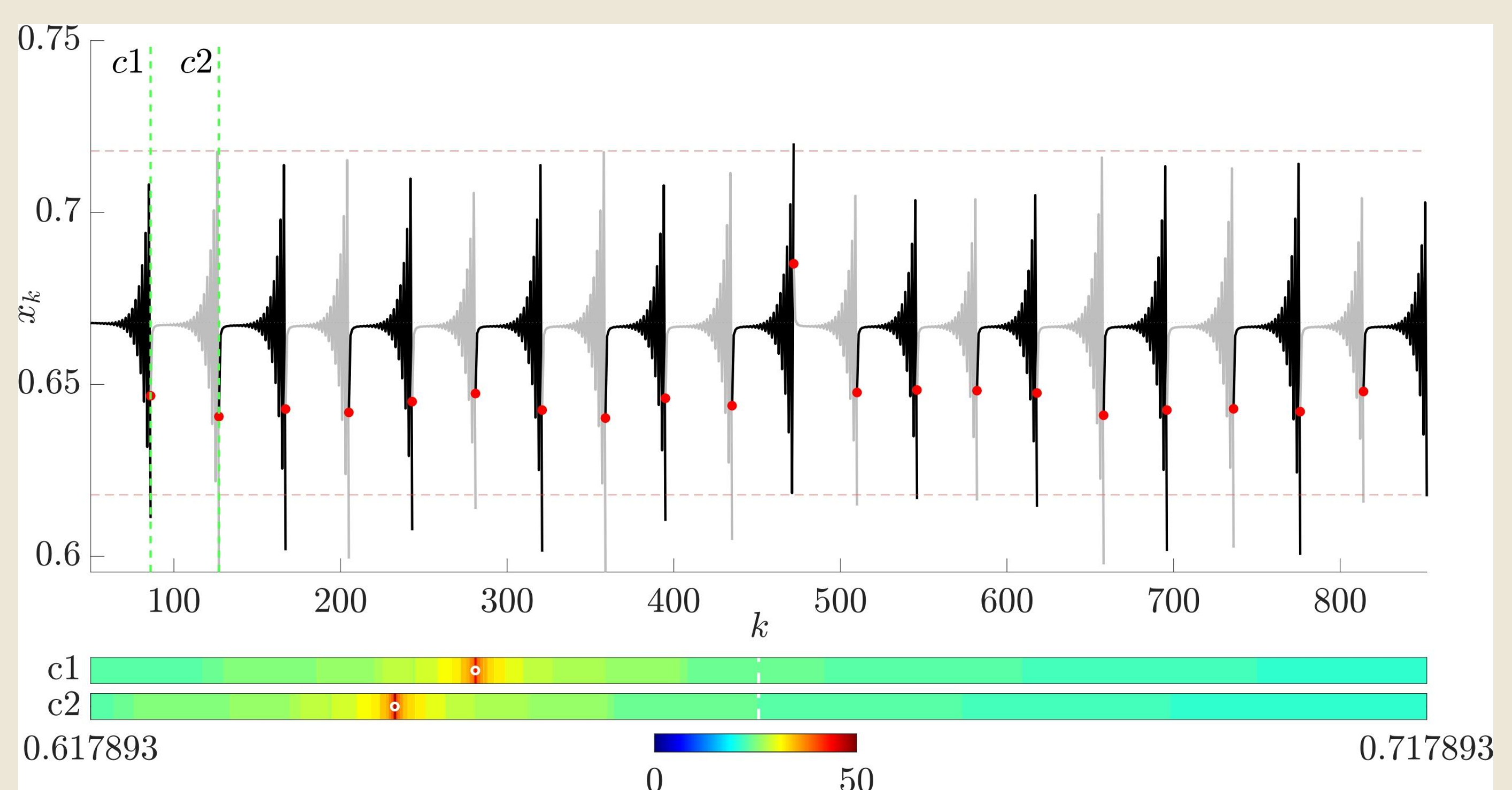


Fig 3. H-rank based impulsive 20 consecutive stabilization attempts. We illustrate that it is possible to continually stabilize the unstable period-1 orbit using the H-rank based impulsive stabilization scheme. Below the stabilization graph is the H-rank pattern that illustrates the first two control attempts and their respective stabilization initial conditions x_k are marked in white circles.

03 Conclusions

□ The naïve stabilization scheme (see Fig. 1) is insufficient to achieve finite-time stabilization of the unstable period-1 orbit because the memory horizon of the FDLM reaches the initial condition inclusively [3].

□ H-rank based impulsive stabilization can achieve finite-time stabilization of the unstable period-1 orbit by finding stabilization initial conditions x_k within high H-rank value bands.

□ Because the unstable period-1 coordinate drifts after every stabilization impulse, we must calculate the H-rank pattern every time we want to stabilize the unstable period-1 orbit. However, we can repeat this algorithm as many times as desired (see Fig. 3). To find out more about this research, refer to [3].

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