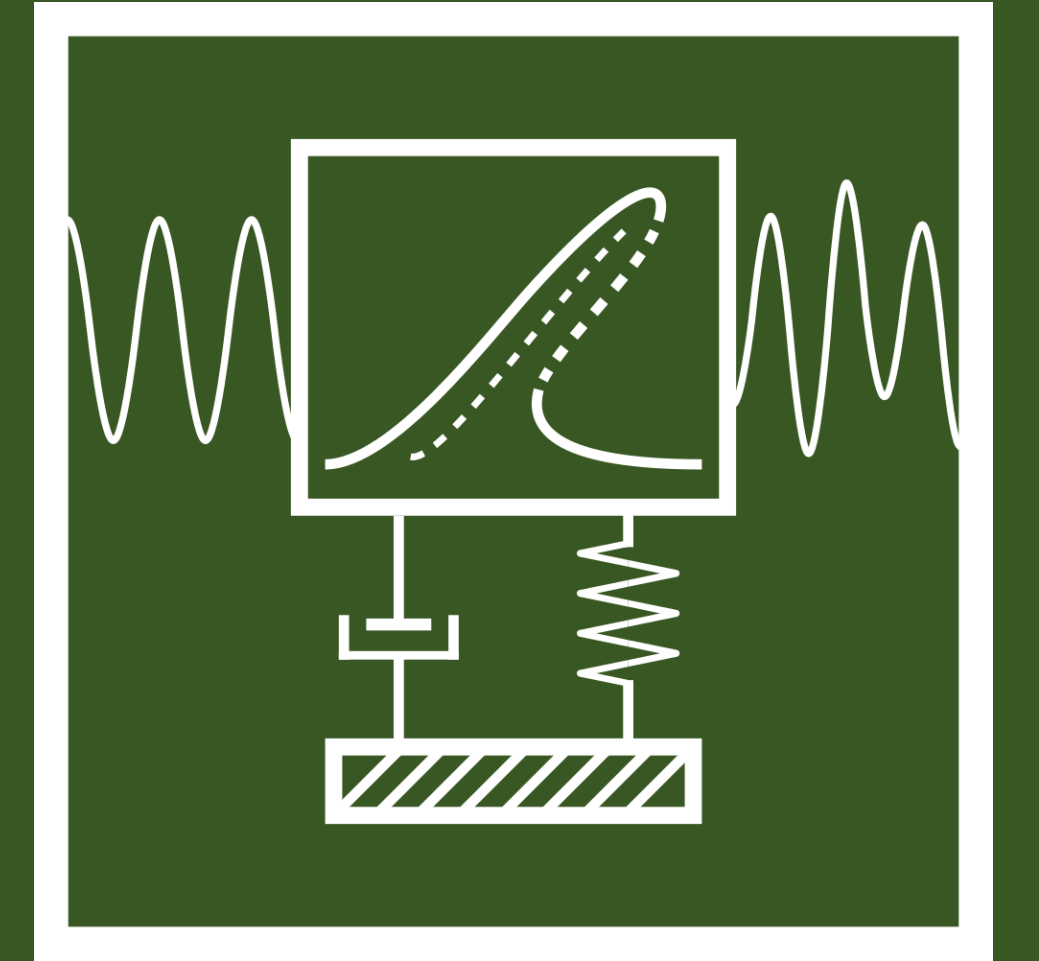


The method of two small parameters for investigating mechanisms with a vibrating base

Genovaitė Zaksienė

Kaunas University of Technology

Corresponding author e-mail: genovaitė.zaksienė@ktu.lt



A

Abstract

The well-known methods such as a small parameter asymptotic, averaging methods in the works by K. Ragulskis and his research were applied and extended creatively. K. Ragulskis and E. Gedgaudaitė suggested the methodology of two small parameters for investigation of nonlinear systems. The method of two small parameters for investigating mechanisms on a vibrating base was offered by K. Ragulskis. One of the parameters evaluates nonlinearity of the conservative system, the other parameter – forcing and dissipative terms. They found out the existing and stability conditions of the synchronization regimes, attraction spheres in the environments of stationary regimes.

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Methods

In this work, new structural schemes were developed and the theory of self-synchronous transmissions of rotational or translational motion is advanced; analytical methods are investigated that can be applied to the solution of strongly nonlinear systems describing the motion of self-synchronous transmissions in the case of combinational synchronization, to the determination of the capture regions of synchronous regimes of motion, and to the study of the dynamics of certain self-synchronous transmissions, which may find wide application in engineering. When considering the combinational self-synchronization of mechanisms, the motion must be described by strongly nonlinear differential equations. K. Ragulskis and E. Gedgaudaitė formed and modified the small-parameter method for the investigation of quasilinear and essentially nonlinear systems. The differential equations of motion are formally divided into slow-motion and quick-motion equations. The quick motion represents an essentially nonlinear conservative system. One of the parameters evaluates the nonlinearity of the conservative system, the other – the forcing and dissipative terms. In practice, self-synchronous systems described by equations of the type

$$F_1(\varphi, \dot{\varphi}, \ddot{\varphi}) = F_2(\varphi, \dot{\varphi}, t), \quad (1)$$

are frequently encountered, where F_1 are the inertial and potential forces and F_2 are the non-potential forces; F_1 is periodic in φ , and F_2 is periodic in φ and t . In order to investigate the dynamics of system Eq. (1), we transform it to the form

$$\ddot{\varphi} + \mu F_1^*(\varphi, \dot{\varphi}, \ddot{\varphi}) = \varepsilon F_2^*(\varphi, \dot{\varphi}, t), \quad (2)$$

where μ is a fictitious small parameter indicating the smallness of the right-hand side compared with the other terms. The generating equation for Eq. (2) has the form

$$\ddot{\varphi} + \mu F_1^*(\varphi, \dot{\varphi}, \ddot{\varphi}) = 0. \quad (3)$$

This equation can be solved by the small-parameter method:

$$\varphi_0 = \varphi_{00} + \mu \varphi_{01}(c_1, t + h) + \mu^2 \varphi_{02}(c_1, t + h) + \dots, \quad (4)$$

where $\varphi_0 = c_1(t + h)$ are determined by the method of quadratures from the equations obtained by substituting series Eq. (4) into Eq. (3) and equating the coefficients at equal powers of μ . Thus, the generating solution is obtained in the form $\varphi_0 = \varphi_0(t + h, c_1, \mu)$, where c_1 and h are arbitrary constants. The period of this solution depends on c_1 . Let c_m denote the value of c_1 at which a synchronous regime exists; then c_m satisfies the equation $T(c_m) = \left(\frac{n}{m}\right) \cdot T$, where m and n are integers and T is the period of the right-hand side. For each value of c_m , a family of solutions $\varphi_0 = \varphi_0(t + h, c_m, \mu)$ depends on h .

Having this generating solution, we seek the solution of Eq. (2) by the method of successive approximations

$$\varphi = \varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots. \quad (5)$$

Substituting Eq. (5) into Eq. (2), we obtain a linear inhomogeneous equation for

$$\varphi_1: \ddot{\varphi}_1 + \mu \left(\partial F_1^* \frac{\partial \varphi, \dot{\varphi}}{\partial \varphi} \right) \cdot \varphi_1 = F_2^*(\varphi_0, \dot{\varphi}_0, t). \quad (6)$$

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Results

The derivative of $F_1^*(\varphi, \dot{\varphi}, \ddot{\varphi})$ is computed for the generating solution $\varphi_0 = \varphi_0(t + h, c_m, \mu)$. The homogeneous equation

$$\ddot{\varphi}_1 + \mu \left(\frac{\partial F_1^*(\varphi, \dot{\varphi}, \ddot{\varphi})}{\partial \varphi} \right) \cdot \varphi_1 = 0, \quad (7)$$

has two particular solutions

$$\varphi_{10} = \frac{\partial \varphi_0(t+h, c_m, \mu)}{\partial h} = x(t + h, \mu), \quad \varphi_{11} = \frac{\partial \varphi_0(t+h, c_1, \mu)}{\partial c_1}.$$

The solution $x(t + h, \mu)$ is periodic with period T . The solution φ_{11} , however, is not periodic, because the periodic function $\varphi_0 = \varphi_0(t + h, c_1, \mu)$ depends on c_1 . The linear combination $\varphi_{12} = h\varphi_{10} - c_m\varphi_{11}$ is only a particular solution of Eq. (7). We transform the solution φ_{12} to the form $\varphi_{12} = tx(t + h, \mu) + y(t + h, \mu)$, where $y(t + h, \mu) = c_m\varphi_{11} - (t + h)x(t + h, \mu)$, and it is a periodic function of period T . The particular solution φ_{12} , together with $\varphi_{10} = x(t + h, \mu)$, is taken as the fundamental solution of the homogeneous Eq. (7).

The general solution of Eq.(6) is sought by the method of variation of the constants of integration:

$$\begin{aligned} \varphi_1 = & C_2 x + C_3 \varphi_{12}, \quad C_2 = C_2(t), \quad C_3 = C_3(t), \\ & D \left\{ x \int_0^t \left(\int_0^t F_2^*(\varphi_0, \dot{\varphi}_0, t) x dt - F_2^*(\varphi_0, \dot{\varphi}_0, t) y - \beta_0 \right) dt \right. \\ & \left. + y \left(\int_0^t F_2^*(\varphi_0, \dot{\varphi}_0, t) x dt - \beta_0 \right) + \alpha_0 x \right\}, \end{aligned}$$

where $D = 1/(x^2 + x\dot{y} - \dot{x}y)$. In this expression the Wronskian determinant $\Delta \neq 0$, by virtue of Liouville's theorem, and α_0, β_0 are the constants of integration. In order for the function φ_1 to be periodic for any α_0 , one must set

$$\int_0^t \left(\int_0^t F_2^*(\varphi_0, \dot{\varphi}_0, t) x dt - F_2^*(\varphi_0, \dot{\varphi}_0, t) y - \beta_0 \right) dt = 0 \quad (8)$$

and

$$4 \int_0^t F_2^*(\varphi_0, \dot{\varphi}_0, t) x dt = 0, \quad (9)$$

From Eq. (9) the constant h is determined, for which a stationary synchronous rotational regime of the type m/n exists. In this case the constant α_0 remains arbitrary, and β_0 is determined from Eq.(8).

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Conclusions

A method with two small parameters is proposed for solving the differential equations describing the motion of self-synchronous mechanisms and transmissions. The principal relationships for the design of self-synchronous mechanisms are obtained, together with the conditions of existence and stability of stationary combinational synchronous regimes, their capture regions, and the dependence of the motion on time.

References

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