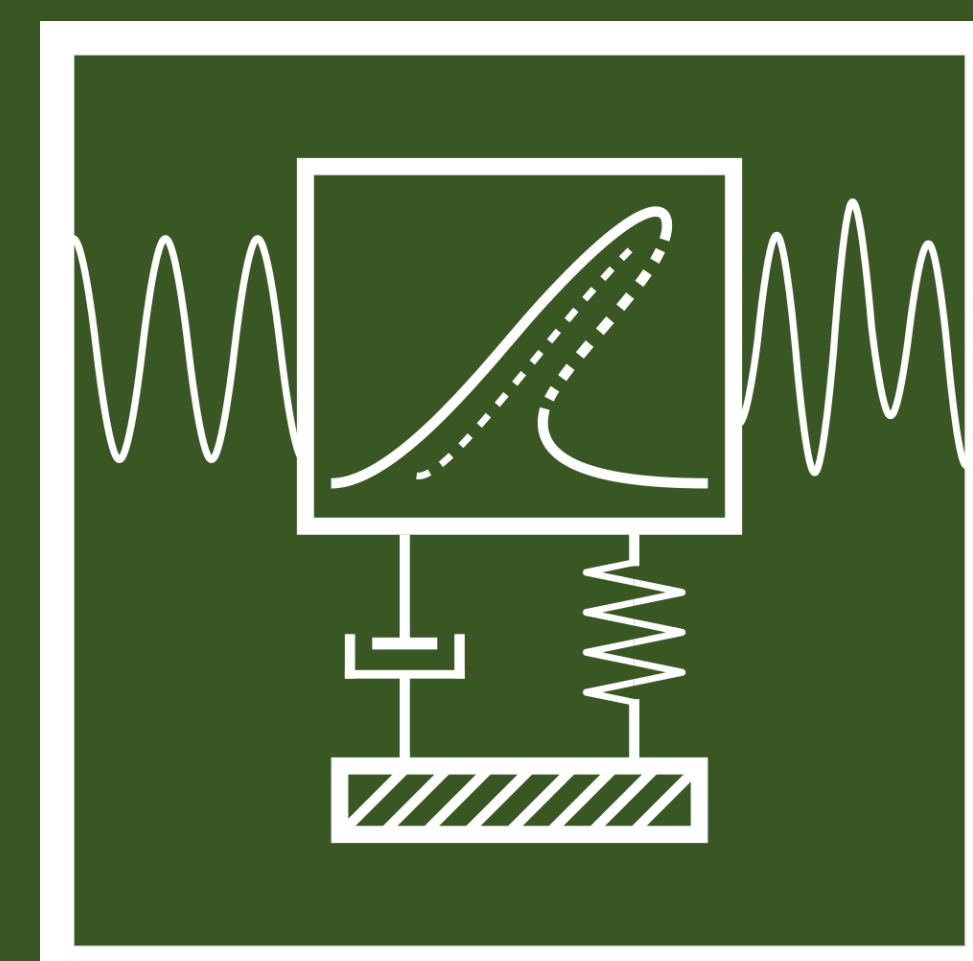


Analysis of Vibrational Systems Using Operator Method

Zenonas Navickas¹, Liepa Bikulciene²

¹ Research Group for Nonlinear Systems, ² Department of applied mathematics, Kaunas University of Technology, Kaunas, Lithuania
Corresponding author e-mail: zenonas.navickas@ktu.lt, liepa.bikulciene@ktu.lt



A Abstract

The operator method is applied to the investigation of several vibrational systems originally proposed by Academician Kazimieras Ragulskis. The study focuses on the analytical treatment of the corresponding differential equations and the search for exact solutions.

01 Methods

Differential equations describing selected vibrational systems are investigated using the operator method. Solutions are represented in operator form, and Hankel matrices and their ranks are used to determine whether exact solutions can be expressed in finite form.

02 Results

$$\begin{aligned}
 & y''_{xx} = P(x, y, y'_x) \quad y(x; s, t)|_{x=v} = s \quad (y(x; s, t))'_x|_{x=v} = t \\
 & p_k(s, t, v) = (D_v + t D_s + P(v, s, t) D_t)^k s \\
 & \begin{vmatrix} p_0 & p_1 & \dots & p_m \\ p_1 & p_2 & \dots & p_{m+1} \\ \dots & \dots & \dots & \dots \\ p_{m-1} & p_m & \dots & p_{2m-1} \\ 1 & \rho & \dots & \rho^m \end{vmatrix} = 0 \qquad \det H_0^{(m)} \neq 0, \\
 & \det H_0^{(m+r)} \equiv 0, \forall r \in N
 \end{aligned}$$

$$y = y(x; s, t, v) = \sum_{k=0}^{+\infty} p_k(s, t, v) \frac{(x-v)^k}{k!}$$



$$\hat{F}(x) = \sum_{r=1}^m \lambda_r \exp(\mu_r x)$$

The analysis shows that the structure and parameters of the considered systems directly influence the rank of the associated Hankel matrices and, consequently, the form of the obtained solutions. For selected parameter configurations, finite-form exact solutions can be identified.

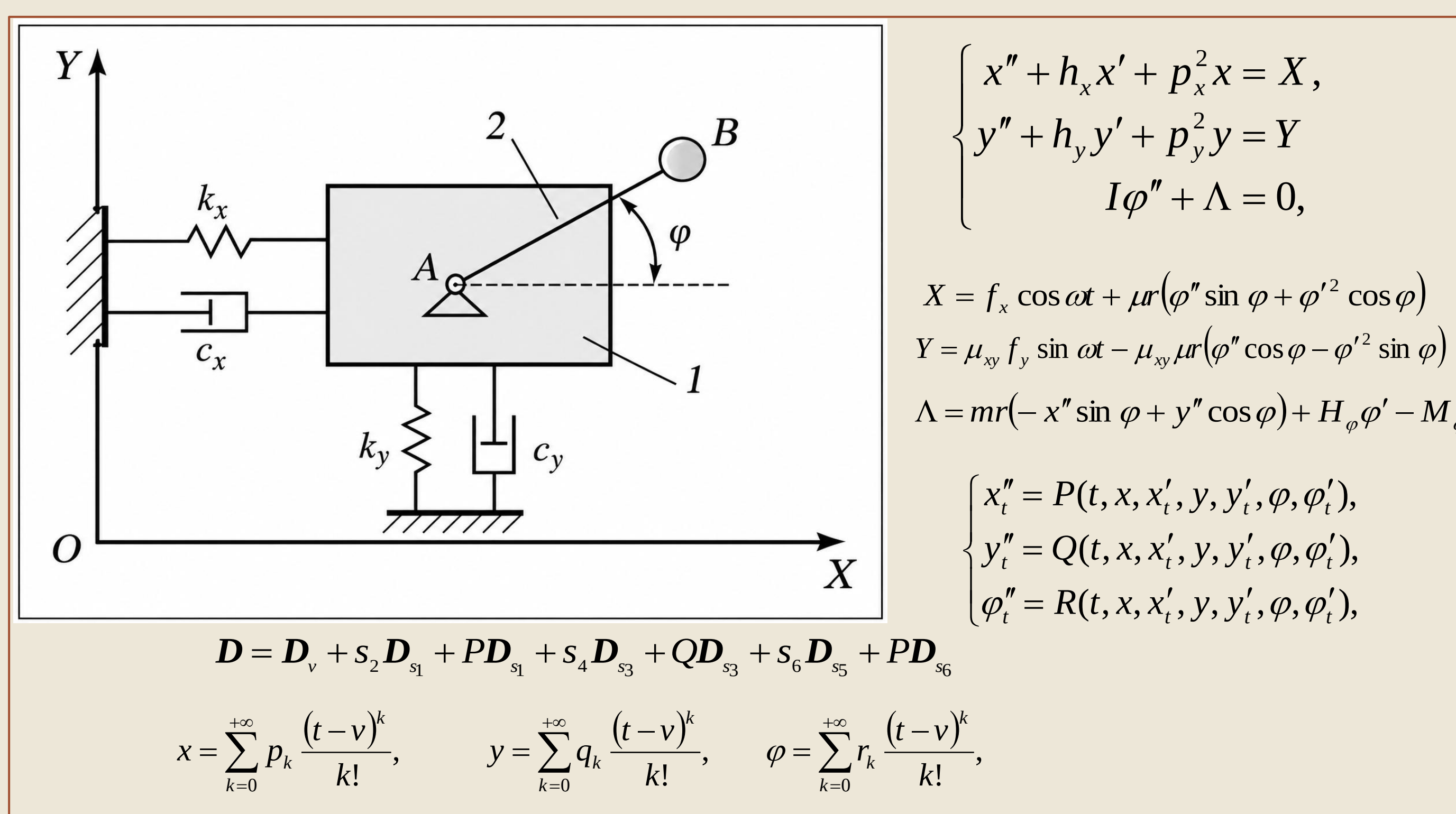


Fig 1. Schematic of the non autonomous model and equations.

03 Discussion

The examples demonstrate how the operator approach provides a systematic way to investigate structurally different vibrational systems and to relate their parameters to the properties of their solutions.

04 Conclusions

The operator method combined with Hankel matrix analysis provides an effective analytical framework for investigating vibrational systems and identifying conditions for obtaining exact solutions.

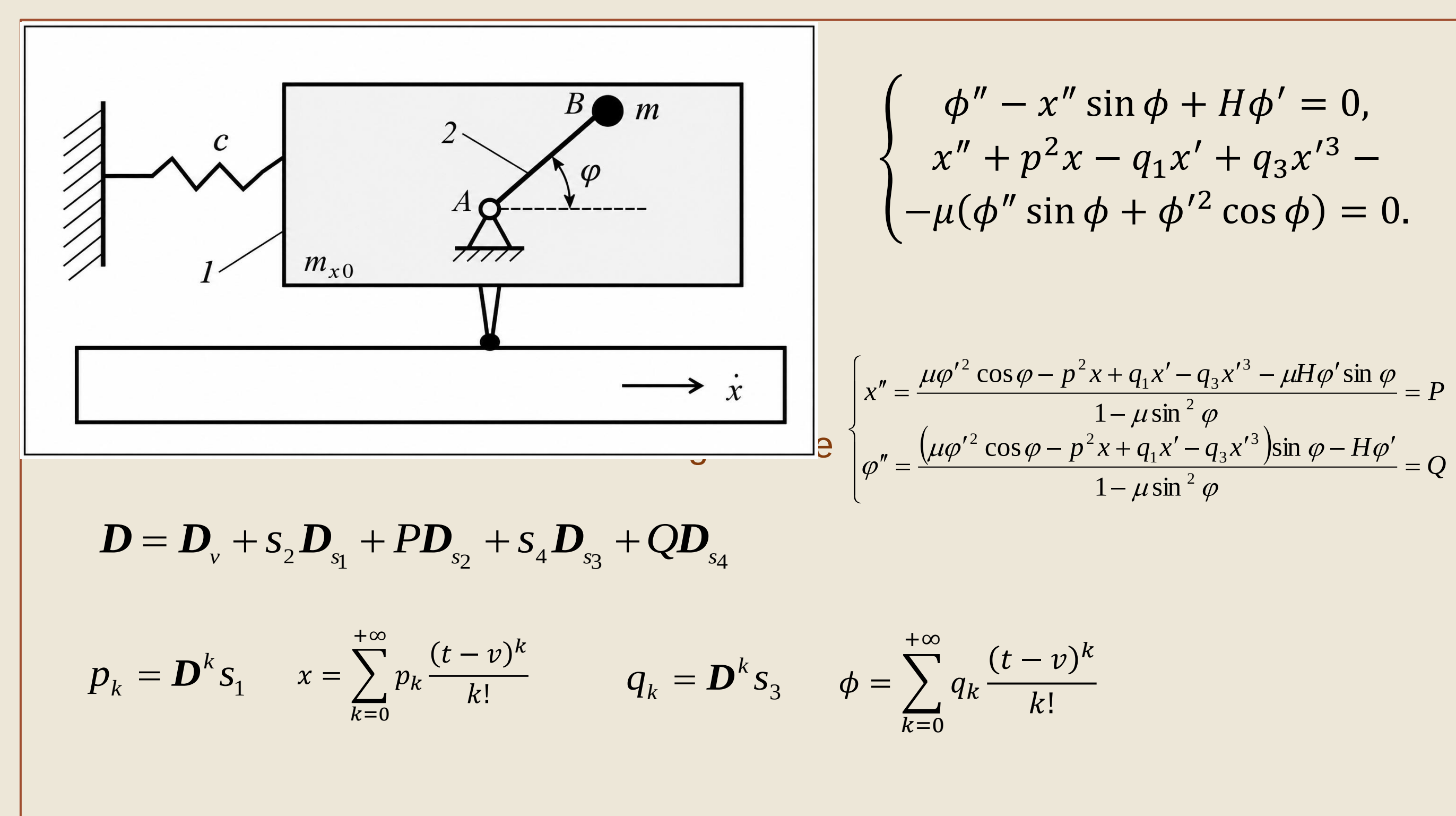


Fig 2. Schematic of autonomous vibrational system and equations

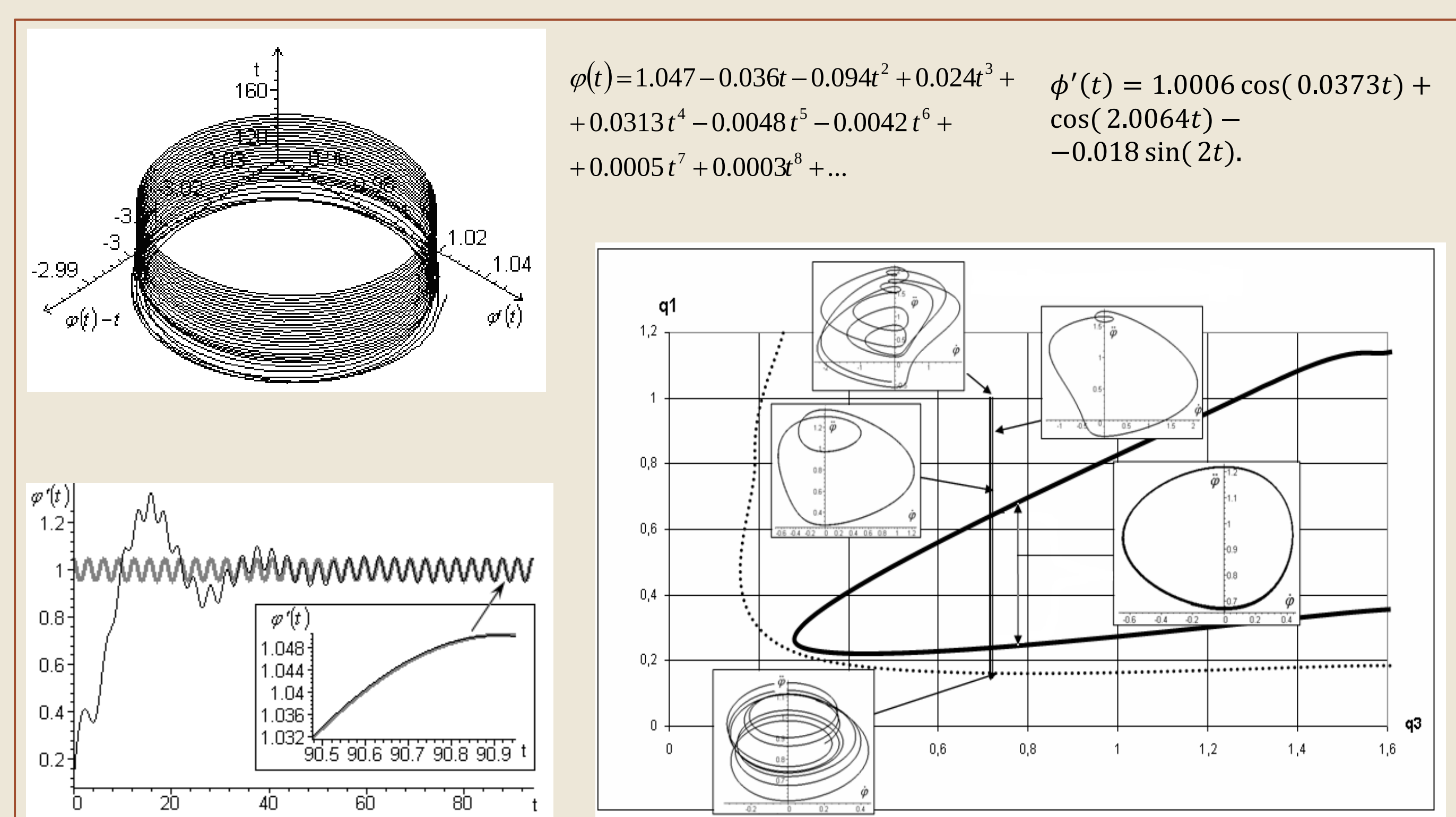


Fig 3. Results for non autonomous and autonomous systems

Acknowledgements

The author gratefully acknowledges Kaunas University of Technology (KTU) for institutional funding support.



References

- [1] K. Ragulskis, Z. Navickas, L. Bikulčienė, "Realization of operator method for mechanical system", Journal of Vibroengineering, Journal of Vibroengineering. Vilnius : Vibromechanika 2002, Vol. 4, no. 1(8), ISSN/eISSN 1392-871, pp. 15-19.
- [2] K. Ragulskis, Z. Navickas, L. Bikulčienė, "Transformation of vibrating process to the rotational motion" ",Vibroengineering 2004 : Proceedings of 5th International Conference, October 14-15, 2004, Kaunas, Lithuania, pp. 44-48.
- [3] Z.Navickas, L. Bikulčienė, M. Rahula, M, Ragulskis, "Algebraic operator method for the construction of solitary solutions to nonlinear differential equations ". Communications in Nonlinear Science and Numerical Simulation, 2013, 18(6), 1374–1389.DOI: 10.1016/j.cnsns.2012.10.009.