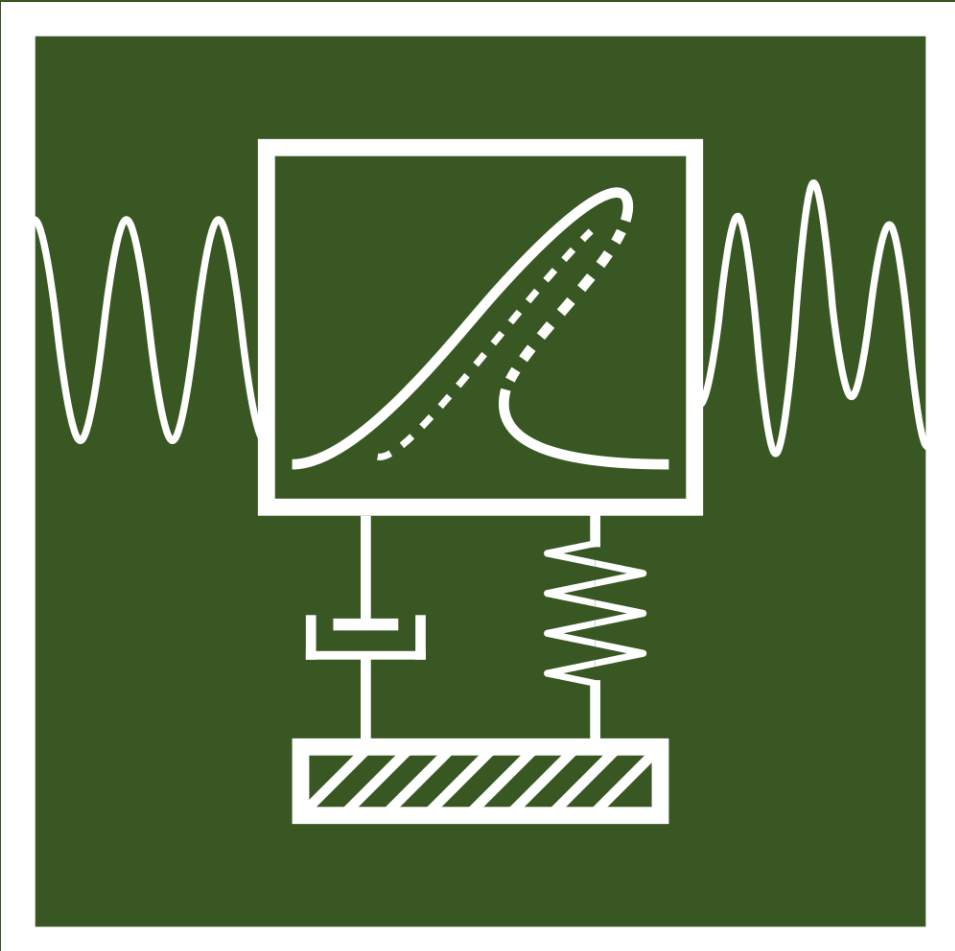


From Vibrotechnika to Digital Twins: Engineering Mathematics at KTU

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A Abstract

Commemorating the centenary of Academician Kazimieras Ragulskis, this work examines the profound influence of his renowned "Vibrotechnika" scientific school on modern engineering education at Kaunas University of Technology (KTU). Ragulskis' legacy demonstrates that engineering breakthroughs in precision mechatronics originate from rigorous mathematical modeling rather than empirical trial-and-error. Today, KTU's pedagogical model transforms engineering training from rote manual computation toward conceptual insight, problem-based learning (PBL), and the application of nonlinear dynamics and modal analysis in developing digital twins. In the era of artificial intelligence and computer-aided engineering, solid mathematical grounding serves as a critical safeguard against numerical instability, ensuring that graduates act as technological innovators rather than mere software operators.

Keywords: Engineering mathematics education, Mathematical modeling, Digital twins, Nonlinear dynamics.

01 Methods

KTU's engineering mathematics curriculum adopts a four-stage integrated pedagogical framework:

- 1. Conceptual Understanding over Routine Computation:** Routine algebraic operations are delegated to computational engines (*MATLAB, Python*), while student focus is directed toward semantic interpretation: derivatives as rates of dynamic change, integrals as accumulated impact, and eigenvalues as resonance boundaries.
- 2. Problem-Based Learning (PBL):** Differential systems and matrix algebra are framed within authentic engineering problems (vibration isolation, UAV flight paths, sensor noise filtering).
- 3. Cognitive Alignment for Gen Z and Alpha:** Procedural memory is engaged through phase-portrait analysis, interactive simulations, and parameter sensitivity testing.

02 Results

Acad. K. Ragulskis' scientific paradigm is embedded into three core engineering laboratory assignments:

- Duffing Nonlinear Resonance:** Analysis of $m\ddot{x} + c\dot{x} + kx + \beta x^3 = F_0 \cos(\omega t)$ reveals response fold-over, bifurcation thresholds, and jump-discontinuities in precision piezoelectric drives.
- MDOF Modal Analysis:** Solving the generalized eigenvalue problem $\det(K - \omega^2 M) = 0$ determines natural frequencies and decouples in-phase and anti-phase modes of dual-mass vibratory platforms.
- Van der Pol Self-Oscillation:** Investigation of $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$ computationally verifies energy-pumping self-synchronization and the formation of an isolated stable limit cycle ($A \approx 2$).

03 Discussion

The rapid adoption of computer-aided engineering (CAE) and artificial intelligence reshapes essential engineering competencies:

- The "Garbage In, Garbage Out" Barrier:** Solvers always produce colorful stress distributions; only a mathematically grounded engineer can detect solver divergence or invalid boundary assumptions.
- Controlling Bifurcations:** In nonlinear systems, infinitesimal parameter shifts can trigger catastrophic resonance collapse. Neural networks cannot foresee these turning points without underlying governing equations.
- Creator vs. Operator:** Grounded in Ragulskis' legacy, KTU ensures graduates create novel architectural mechanisms rather than remaining mere consumers of commercial software.

04 Conclusions

- Academician K. Ragulskis' foundation of nonlinear dynamics remains integral to contemporary engineering training at KTU.
- Shifting to conceptual modeling and problem-based learning bridges fundamental mathematics with the synthesis of industrial digital twins.
- In the era of AI and automated simulation, mathematical rigor provides the decisive competency for auditing models and preventing catastrophic failures.

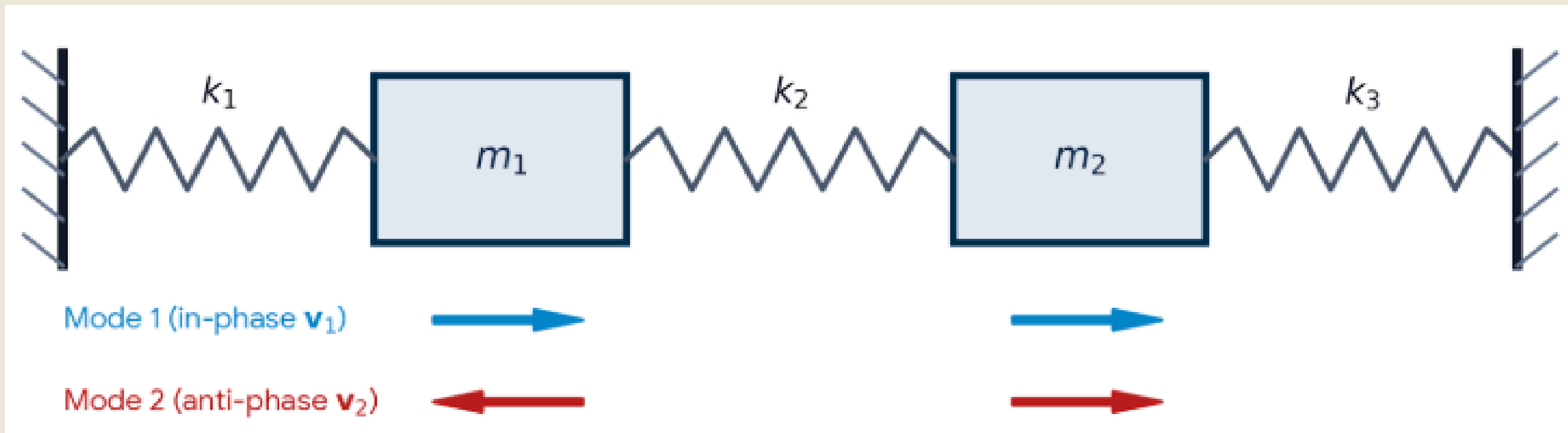


Fig 2. Phase-space trajectory and modal displacement vectors of a 2-DOF resonant vibratory platform.

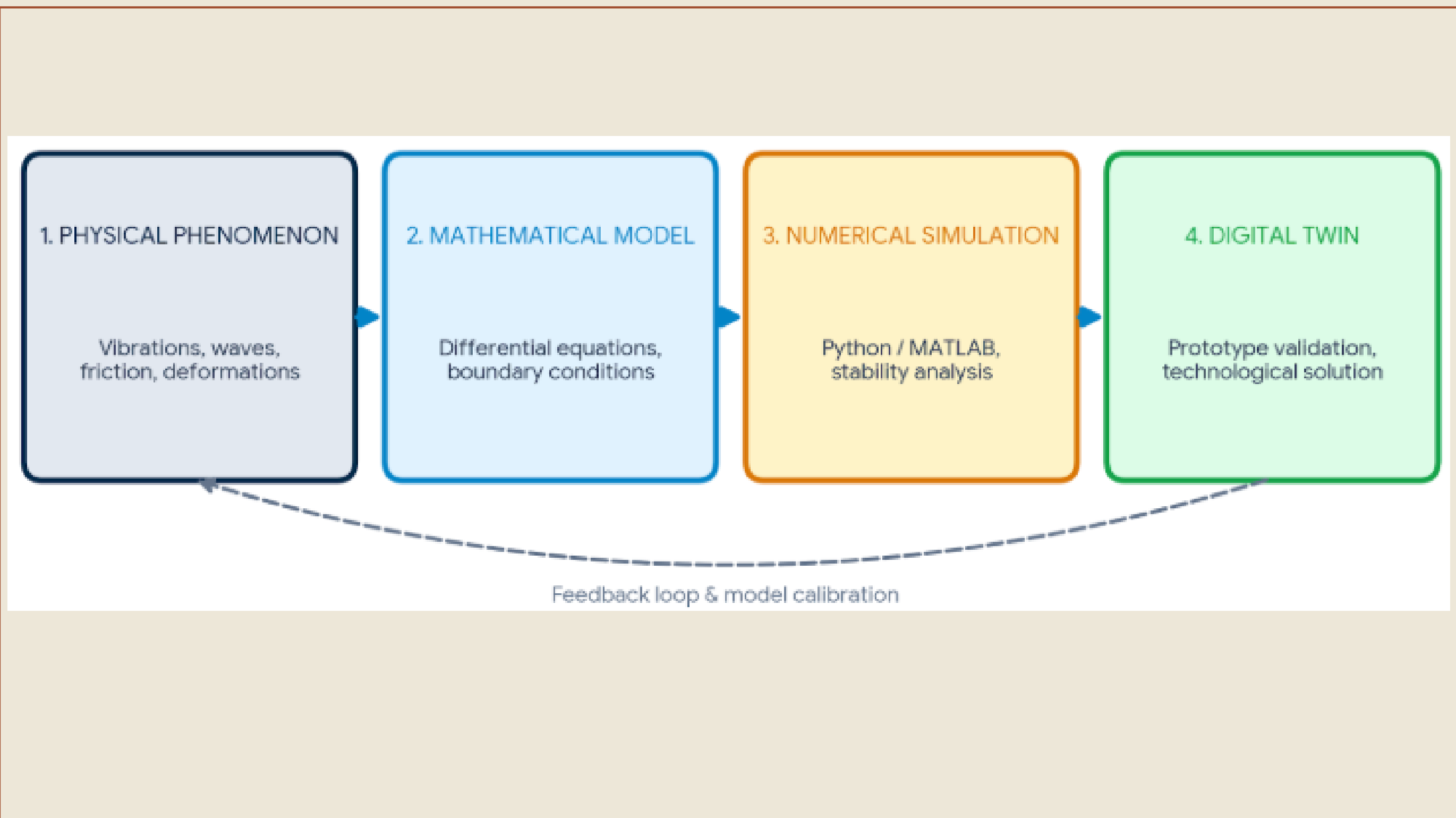


Fig 1. Schematic of the engineering learning cycle at KTU.

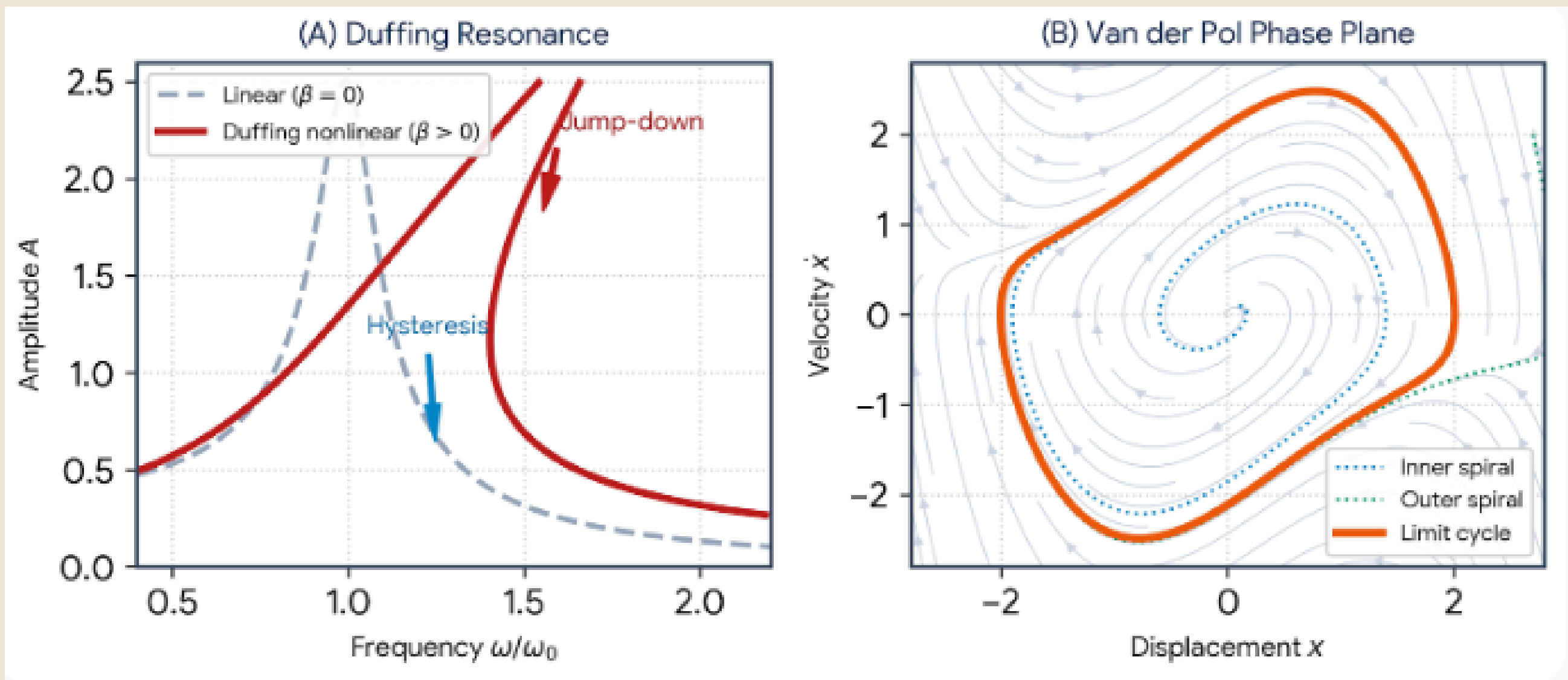


Fig 3. Numerical results: Duffing jump phenomenon under frequency sweep and Van der Pol phase-space limit cycle.

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